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Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2018

Third Semester

Core Course (3)—CALCULUS

[Common for Model I, Model II Mathematics and B.Sc. Computer Applications]

(2013—2016 Admissions)

Time : Three Hours Maximum Marks : 80

Part A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Find n^{th} derivative of $\cos(ax+b)$.
2. State Maclaurin's theorem.
3. Check whether $f(x, y) = x^2 + y^2$ is a homogeneous function.
4. Define envelope of a function.
5. Define regular points.
6. If $z = e^{xy}$, find $\frac{\partial z}{\partial y}$.
7. Define length of a parametric curve.
8. State Pappus theorem for volumes.
9. Describe Cylindrical co-ordinates.
10. Define the double integral of a function of two variables over a bounded region in the co-ordinate plane.

(10 × 1 = 10)

Turn over

Part B

Answer any **eight** *questions.*
Each question carries 2 marks.

11. Find $\frac{d^2y}{dx^2}$ where $x = \sec^2 t$ and $y = \tan^2 t$.

12. Find $\frac{dw}{dt}$ if $w = xy + z$ $x = \cos t$, $y = \sin t$, $z = t$. What is the derivatives value at $t = 0$?

13. Show that function $f(x, y) = e^{-2y} \cos 2x$ satisfies Laplace equation.

14. Verify Euler's theorem for $u = x^3 - y^3 + 3x^2 y$.

15. Find the center of curvature at the point $\left(\frac{a}{4}, \frac{a}{4}\right)$ of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$.

16. Find the asymptotes of $3x^2 - 5xy - 2y^2 + 17x + y + 14 = 0$.

17. Find a spherical co-ordinate equation for the cone $z = \sqrt{x^2 + y^2}$.

18. Find the evolute of the parabola $x^2 = 4ay$.

19. Evaluate the integral $\int_0^1 \int_0^{1-y} \int_0^2 dx\, dz\, dy$.

20. Find the limits of integration for integrating (r, θ), over the region R that lies inside the cardioid $r = 1 + \cos\theta$ and outside the circle $r = 1$.

21. Evaluate $\iint_R e^{x^2 + y^2}\, dx\, dy$, where R is the semicircular region bounded by the x-axis and the curve $y = \sqrt{1 - x^2}$.

22. Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}$, $1 \le x \le 2$ about the x-axis.

$(8 \times 2 = 16)$

Part C

Answer any **six** *questions.*
Each question carries 4 marks.

23. Find the n^{th} derivative of $y = e^{ax}(\sin bx + c)$.

24. Show that the evolute of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ is another cycloid.

25. Find the area of the region cut from the first quadrant by the curve $x^{\frac{1}{2}} + y^{\frac{1}{2}} = a^{\frac{1}{2}}$.

26. Find the co-ordinates of center of curvature at any point of the parabola $x^2 = 4ay$ and also find its evolute.

27. Evaluate $\int_0^{4a}\int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dydx$ by changing the order of integration.

28. Find the volume of the solid generated by revolving the region in the first quadrant bounded above by the curve $y = x^2$, below by the x-axis, and on the right by the line $x = 1$, about the line $x = -1$.

29. Find the length of the curve $y = \frac{4\sqrt{2}}{3}x^{\frac{3}{2}} - 1$, $0 \le x \le 1$.

30. Find the area of the region cut from the first quadrant by the cardioid $r = 1 + \sin\theta$.

31. Find the volume of the solid generated when the region enclosed by the curve $y = \frac{x^2}{9 - x^2}$, $y = 0, x = 0, x = 2$, is revolved about the x-axis.

$(6 \times 4 = 24)$

Part D

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) Find the volume of the solid cut from the first octant by the surface $z = 4 - x^2 - y$.

(b) Find the area of the region R enclosed by the parabola $y = x^2$ and the line $y = x + 2$.

33. (a) Find the area of the region enclosed by the positive x-axis and spiral $r = \frac{4\theta}{3}$, $0 \le \theta \le 2\pi$.

(b) Find the envelope of the curve $y^2 = 4a(x - a)$.

34. Find the area enclosed by the lemniscate $r^2 = 4\cos 2\theta$.

35. Find the volume of the region D enclosed by the surfaces $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$.

$(2 \times 15 = 30)$